Reporting Measurements and Uncertainties, Significant Digits, and Rounding

Jack Merrin

February 10, 2017

1 Summary

In this lesson, we talk about the number of significant digits used to report measurements and uncertainties. There are conventional rules for the number of significant digits in a number. Experts find that the actual number of significant digits in a measurement is a function of how many significant digits in the uncertainty are used. The best practice is to use two significant digits of uncertainty and to match the precision of the measurement and the uncertainty. Reported measurements should be recorded as a value plus or minus another value, with parenthesis indicating the error in the last digits, or with scientific notation. For measurements that end in zero or zeros, there may be different conventions for indicating or calculating the number of significant digits. An alternative method is to use scientific notation, where all the digits are significant. Calculation of derived quantities from properly reported measurements and uncertainties also make new reportable quantities. Using two significant digits of uncertainty minimizes potential rounding errors.

2 Correctly reporting measurements and scientific notation

The best way to learn how to correctly report measurements and uncertainties is to see some examples. It would be correct to write any of the following.

$$57.91 \pm 0.46 \quad 57.91(46)$$

$$(5.791 \pm 0.046) \times 10^1 \quad 5.791(46) \times 10^1$$

We have introduced here scientific notation. A number in scientific notation is of the form $A \times 10^B$ where $1 \leq A < 10$ and B is an integer. The two digits in parenthesis represent the uncertainty in the last two digits of the measurement. It would be incorrect to report any of these because the precision does match.

$$10.47 \pm 0.022 \qquad 10.4 \pm 0.22 \qquad (1.047 \pm 0.22) \times 10^1$$

I would be very suspicious of the data analysis of any scientific report or book that makes this basic error.

3 Rules for significant digits

The classic rules for the significant digits in a number are as follows. All non-zero digits are significant. In 1204 1, 2, and 4 are significant for sure. All zeroes between non-zero digits are significant. In 1204, the 0 is also significant. Leading zeros are not significant. 0.012 has only two significant digits Trailing zeroes after the decimal point are significant. 12.0 and 0.0400 both have three significant digits. Trailing zeroes without a decimal point are not significant. 1000 has only one significant digit.

4 Keeping 2 significant digits

I see a lot of books that give the rule, only use one significant digit of uncertainty. Unfortunately, this is not what the professionals are doing. If you go to the NIST or CODATA website that reports the fundamental constants. They are all given with two digits of uncertainty. We should try to emulate their example.

There is in fact only one rule. Round to two significant digits of uncertainty and match the precision of the measurement by rounding. For example, if we have (4200 ± 43) or (4200 ± 1500) then we see that 4200 has a different number of significant digits in each instance. In the first case, 4200 has four significant digits, it just happens to end in two zeros. In the second case, 4200 has only two significant digits because the uncertainty has two significant digits and the precision must match. It would be incorrect to write (4212 ± 1500). The digits 12 are insignificant in this instance and should be changed to zeros.

5 Rules for Rounding

We also do not write $(671941.283962 \pm 1.56932)$. Although the precision does match, one cannot justify actually knowing that many digits of uncertainty. Such an expression can be fixed however and written as (671941.3 ± 1.6). Notice some rounding is involved. The rules for rounding are as follows. Consider the last two digits of the number as xy. If y is 0, 1, 2, 3, or 4 keep x. If y is 5, 6, 7, 8, or 9 then add one to x. Either remove y or replace y with zero depending on the situation.

Rounding the uncertainty is easy $(0.0445 \to 0.045)$, $(0.0444 \to 0.44)$, $(12.2 \to 12)$, $(12.9 \to 13)$, $(2501 \to 2500)$, $(2555 \to 2600)$, $(0.999 \to 1.0)$, and $(99.64 \to 100)$. Notice that 100 still only has two significant digits the first 1 and the first 0. For example, we could write (145.32 ± 99.9921) as (150 ± 100).

6 How to do the calculation?

In practice, we keep all the digits for our calculation but then do the rounding at the end. We will usually have an error propagation formula or a method to calculate the spread of our data determining the uncertainty. We then round this to two significant digits. Then we can match our calculator value of the measurement. For example, $(4.221242 \pm 0.43412) \rightarrow 4.22(43)$ If you feel there is ever any ambiguity then use scientific notation to indicate your significant digits.

7 Uncertainty in derived quantities

If we wanted to add two numbers with uncertainty like (12.1 ± 1.0) and (11.8 ± 1.0) then we would get (23.9 ± 1.4). The uncertainty is calculated as $\sqrt{1.0^2 + 1.0^2}$ in this situation. We will learn this later using propagation of error. If we only used one significant digit there would be a major 41 percent rounding error, $(\sqrt{2} = 1)$. This is one of the main reasons for using (at least) two significant digits. It is not necessary to keep an uncertainty to one part in a thousand by convention.

8 Exact quantities

Certain fundamental physical constants or mathematical constants are defined so you don't consider their significant digits. They have no uncertainty. You can pretend they have an infinite number of significant digits. If you are working with the speed of light then plug into your calculator

$$c = 299792458\,\mathrm{ms}^{-1}$$

If you are calculating the area of a circle and your radius is 4.34 then you should use $(\pi = 3.1416)$ just to have more digits than the number you are working with. You can use more digits but it won't affect the rounding in the end.

Another example, where significant digits do not come into play, is conversion factors. The definition of an inch is that 25.4 mm equals one inch. This doesn't mean that all your calculations have to come out with 3 significant digits. Keep using the methods that are outlined in this lesson.